

**Math 115**  
**Spring 2019**  
**Lecture 24**

?  $a^2 + b^2 = c^2$  ?  
 $y = mx + b$  ?  $d = rt$

Ch. 6 Rational Expressions

Polynomial

Polynomial

$$\frac{12x^3y^5}{3xy^8}, \quad \frac{3x-5y}{9x-15y}, \quad \frac{4x^2-25}{x^2+3x+15}, \quad \frac{x^2-4x+3}{x^2-9}$$

How to reduce rational expression

1) Factor numerator & denominator completely.

2) Cross-out common factors.

Simplify:

$$\frac{3 \cancel{12} x^3 y^6}{\cancel{4} x y^8} = \frac{3 x^2}{y^2}$$

$$\frac{6x + 10}{9x + 15} = \frac{2(3x + 5)}{3(3x + 5)} = \frac{2}{3}$$

$$\frac{x^2 - 25}{x^2 + 7x + 10} = \frac{(x-5)(x+5)}{(x+5)(x+2)} = \frac{x-5}{x+2}$$

Simplify

$$1) \frac{\cancel{5} \cancel{25} x^{12} y^3}{\cancel{10} x^8 y^{10}} = \frac{-5 x^4}{2 y^7}$$

$$2) \frac{4x - 5y}{16x - 20y} = \frac{1(4x - 5y)}{4(4x - 5y)} = \frac{1}{4}$$

$$3) \frac{2x^2 + 3x - 5}{x^2 + 3x - 4} = \frac{(x-1)(2x+5)}{(x+4)(x-1)} = \frac{2x+5}{x+4}$$

$$2x^2 + 3x - 5 = 2x^2 - 2x + 5x - 5$$

5, -2      -10

$2x(x-1) + 5(x-1)$

Excluded Values: These are the Values that make the denominator equal to Zero.

How to Find excluded Values:

1) Make denominator = 0

2) Solve.

Find excluded Values:

$$\frac{2x-1}{x-4}$$

$$x-4=0$$

$$\boxed{x=4}$$

E.V. 4

Find excluded Values:

$$\frac{x-2}{2x+5}$$

$$2x+5=0$$

$$2x=-5$$

$$x=-\frac{5}{2}$$

E.V.  $-\frac{5}{2}$

$$\frac{7x}{(x-8)(x+6)}$$

$$(x-8)(x+6)=0$$

use Z.F.P.

$$x-8=0 \quad \text{or} \quad x+6=0$$

$$\boxed{x=8}$$

or

$$\boxed{x=-6}$$

E.V. 8, -6

$$\frac{3x+10}{x^2-10x+21}$$


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$$\frac{x^2-8x+12}{x^2-100}$$

$$x^2-100=0$$

$$(x+10)(x-10)=0$$

$$x+10=0 \quad x-10=0$$

$$\boxed{x=-10} \quad \boxed{x=10}$$

$$x^2-10x+21=0$$

$$(x-7)(x-3)=0$$

by Z.F.P.

$$x-7=0 \quad \text{or} \quad x-3=0$$

$$\boxed{x=7} \quad \boxed{x=3}$$

E.V. 3 & 7

E.V.  $\pm 10$

$$\frac{7x+5}{2x^2-3x-14}$$

To find E.V.

Denom. = 0  $\rightarrow$  Then Solve

$$2x^2-3x-14=0$$

Let's use Quadratic Formula

$a=2 \quad b=-3 \quad c=-14$

$$b^2-4ac = (-3)^2 - 4(2)(-14) = \boxed{121}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-3) \pm \sqrt{121}}{2(2)} = \frac{3 \pm 11}{4}$$

$$x = \frac{3+11}{4} = \frac{14}{4} = \boxed{\frac{7}{2}}$$

$$x = \frac{3-11}{4} = \frac{-8}{4} = \boxed{-2}$$

E.V.  $-2$  &  $7/2$

Find excluded values

①  $\frac{x}{2x+9}$

$2x+9=0$

$2x=-9$

$x=-\frac{9}{2}$

E.V.

$-\frac{9}{2}$

②  $\frac{x-1}{(x-7)(3x+5)}$

$x-7=0$

$x=7$

$3x+5=0$

$x=-\frac{5}{3}$

E.V.

$-\frac{5}{3}, 7$

③  $\frac{2x}{x^2-5x+6}$

$(x-2)(x-3)=0$

$x-2=0$

$x=2$

$x-3=0$

$x=3$

E.V.  $2 \neq 3$ 

$3x^2-5x-8=0$

$a=3 \quad b=-5 \quad c=-8$

$b^2-4ac=(-5)^2-4(3)(-8)=121$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{5 \pm 11}{6}$

$x = \frac{5+11}{6}$

$x = \frac{16}{6}$

$x = \frac{8}{3}$

$x = \frac{5-11}{6}$

$x = \frac{-6}{6}$

$x = -1$

E.V.  $-1 \neq \frac{8}{3}$ 

How to multiply rational Expressions:

① Factor everything Completely

② Cross-out the Common Factors  
Vertically or diagonally

$$\frac{x^2-9}{2x+6} \cdot \frac{2x+4}{x^2-2x-3} = \frac{\cancel{(x-3)}\cancel{(x+3)}}{\cancel{2}\cancel{(x+3)}} \cdot \frac{\cancel{2}(x+2)}{(x+1)\cancel{(x-3)}}$$

$$= \boxed{\frac{x+2}{x+1}}$$

multiply:

$$\frac{x^2 + 5x + 6}{x^2 + 7x + 12} \cdot \frac{x^2 - 16}{x^2 - 2x - 8}$$

$$= \frac{\cancel{(x+3)}(x+2)}{\cancel{(x+4)}(x+3)} \cdot \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}(x+2)} = \boxed{\frac{x+2}{x+2}} = 1$$

multiply

$$\frac{2x^2 + 3x - 5}{2x^2 + 5x - 7} \cdot \frac{x^2 - 100}{x^2 - 9x - 10}$$

$$= \frac{(2x+5)\cancel{(x-1)}}{(2x+7)\cancel{(x-1)}} \cdot \frac{(x+10)\cancel{(x-10)}}{(x+1)\cancel{(x-10)}}$$

$$= \boxed{\frac{(2x+5)(x+10)}{(2x+7)(x+1)}}$$

How to divide rational expressions:

① change  $\div$  to  $\cdot$ .

② multiply by reciprocal.

③ Factor, and cross-out common factors.

$$\frac{2x+8}{x^2-25} \div \frac{x^2-16}{3x+15} = \boxed{\frac{6}{(x-5)(x-4)}}$$

$$\frac{2x+8}{x^2-25} \cdot \frac{3x+15}{x^2-16} = \frac{\cancel{2(x+4)}}{\cancel{(x+5)}(x-5)} \cdot \frac{3\cancel{(x+5)}}{\cancel{(x+4)}(x-4)}$$

Divide

$$\frac{x^2-4}{x^2+4x+4} \div \frac{x^3-8}{2x-4}$$

$$= \frac{x^2-4}{x^2+4x+4} \cdot \frac{2x-4}{x^3-8} = \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x+2)} \cdot \frac{2\cancel{(x-2)}}{\cancel{(x-2)}(x^2+2x+4)}$$

$$A^3 - B^3$$

$$x^3 - 2^3 = (x-2)(x^2+2x+4)$$

$$= \boxed{\frac{2(x-2)}{(x+2)(x^2+2x+4)}}$$

Simplify:  $\frac{11}{10} - \frac{3}{10} = \frac{11-3}{10} = \frac{8}{10}$

$$= \frac{4}{5}$$

Simplify:  $\frac{x}{x^2-9} - \frac{3}{x^2-9}$

$$= \frac{x-3}{x^2-9} = \frac{1(\cancel{x-3})}{(\cancel{x-3})(x+3)}$$

$$= \frac{1}{x+3}$$

Simplify  $\frac{x^2}{x^3+125} - \frac{25}{x^3+125}$

$$= \frac{x^2-25}{x^3+125} \quad \begin{matrix} \nearrow x^3+5^3 \\ \searrow \end{matrix}$$

$$= \frac{(\cancel{x+5})(x-5)}{(\cancel{x+5})(x^2-5x+25)} = \frac{x-5}{x^2-5x+25}$$

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$$\frac{3x+5}{4x^2-25} - \frac{x}{4x^2-25} = \frac{3x+5-x}{4x^2-25}$$

$$= \frac{2x+5}{4x^2-25}$$

$$= \frac{1(\cancel{2x+5})}{(\cancel{2x+5})(2x-5)} = \frac{1}{2x-5}$$